Unit-IV: Digital Modulation Scheme

Geometric Representation of signals - Generation, detection, PSD & BER of Coherent BPSK, BFSK & QPSK - QAM Carrier Synchronization - Structure of Non-coherent Receivers - Principle of DPSK.

Digital Modulation - is the process of changing the characteristics of carrier wave in accordance binary message signal.

Types:-

1) Coherent Modulation/demodulation
2) Non-Coherent Modulation/demodulation.

Coherent Modulation: The term coherent represents the synchronization of carrier frequency between transmitter and receiver. It means that the receiver has knowledge about the carrier frequency at the tran...

Types:

1) Amplitude shift keying (ASK)
2) Phase shift keying (PSK)
3) Frequency shift keying (FSK).

Non-Coherent Modulation/Detection:

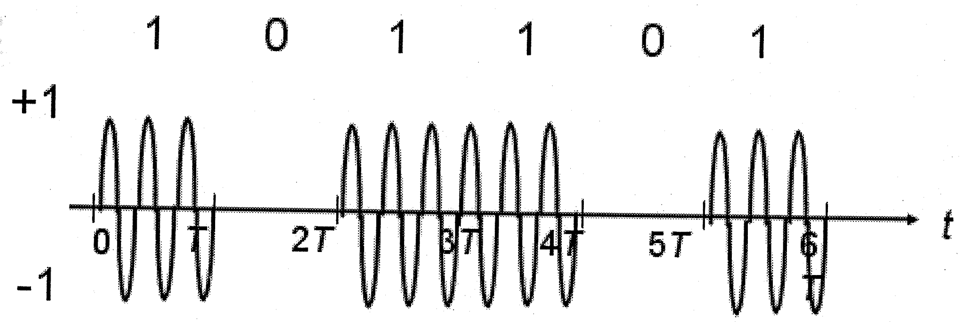
The receiver doesn't have any knowledge about the carrier frequency at the transmitter.

Designal goals of Digital Modulation schemes:

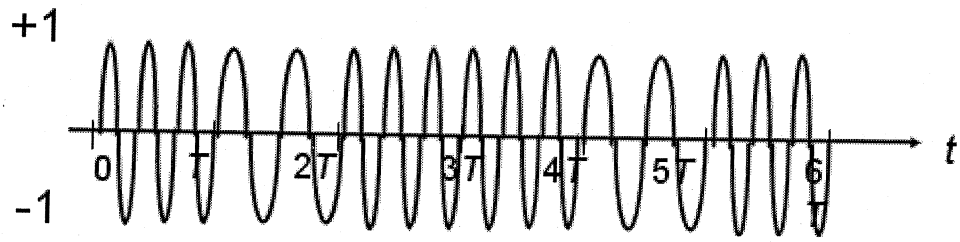
1) Minimum Data rate
2) Minimum Probability of Error.
3) Minimum transmission Power.
4) Maximum channel BW utilization.
5) Maximum resistance interfering signals.
6) Minimum circuit complexity.

REPRESENTATION OF DIGITAL SIGNAL

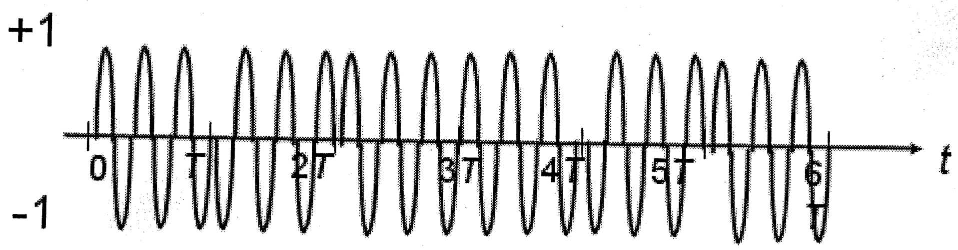
AMPLITUDE SHIFT KEYING



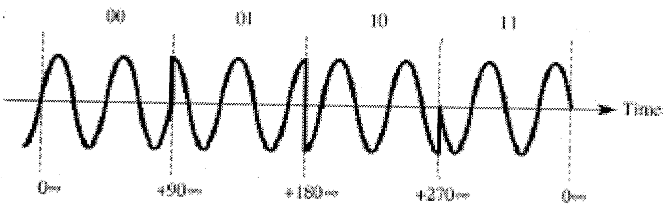
BINARY FREQUENCY SHIFT KEYING



BINARY PHASE SHIFT KEYING



QUADRATURE PHASE SHIFT KEYING



Coherent Binary PSK:-

Coherent refers coherent detection (i.e) the exact replica of input signals are available at the receiver since the receiver has the exact knowledge of the carrier wave's phase reference.

Disadv: Rx Circuit complexity.

Adv :- Less bit error rate.

Binary PSK Transmitter:

Binary PSK signals have a constant envelope and symbol '1' is represented with 0° phase shift and symbol '0' is represented with 180° phase shift of sinusoidal carrier wave signals. It is defined by antipodal signals. Antipodal signals is a pair of sinusoidal wave that differ only in a relative phase shift of 180°, as given in the equation.

$$\left.\begin{aligned} s_1(t) &= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \\ s_2(t) &= \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) \\ &= -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t). \end{aligned}\right\} \quad \text{——— (1)}$$

where E_b – Transmitted signal energy per bit

f_c – carrier frequency $= n_c/T_b$

n_c – number of cycles of carrier wave in each transmitted bit duration.

T_b – bit duration

~~Radom~~ basis fn of unit energy is given as

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b \quad (2)$$

∴ eqn (1) may be rewritten as.

$$\left.\begin{aligned} S_1(t) &= \sqrt{E_b}\,\phi_1(t) \\ S_2(t) &= -\sqrt{E_b}\,\phi_1(t) \end{aligned}\right\} \quad 0 \le t \le T_b.$$

Thus I/p binary wave in polar form with amplitude levels of $\sqrt{E_b}$ & $-\sqrt{E_b}$ resply and are multiplied by ϕ_1 is tone — symbol '1' & 0 are rep by

In Binary PSK, a signal space is one dimensional $(N=1)$; with two message points $(M=2)$.

The coordinates of the message points

$$S_{11} = \int_0^{T_b} s_1(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \sqrt{E_b}\,\phi_1(t)\,\phi_1(t)\,dt$$

$$= \sqrt{E_b}\int_0^{T_b} \phi_1^2(t)\,dt = \sqrt{E_b}\int_0^{T_b}\left[\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t\right]^2 dt$$

$$= \sqrt{E_b}\cdot\frac{2}{T_b}\int_0^{T_b}\cos^2(2\pi f_c t)\,dt$$

$$= \sqrt{E_b}\,\frac{2}{T_b}\int_0^{T_b}\frac{(1+\cos 2(2\pi f_c t))}{2}\,dt$$

$$= \sqrt{E_b}\,\frac{2}{T_b}\left[\int_0^{T_b}\frac{dt}{2} + \int_0^{T_b}\frac{\cos(4\pi f_c t)}{2}\,dt\right]$$

$$= \sqrt{E_b}\,\frac{2}{T_b}\left[\frac{t}{2}\Big|_0^{T_b} + \frac{1}{2}\left(+\frac{\sin 4\pi f_c t}{4\pi f_c}\right)_0^{T_b}\right]$$

$$= \sqrt{E_b}\,\frac{2}{T_b}\left[\frac{T_b}{2} + \frac{\sin 4\pi \frac{n_c}{\cancel{T_b}} \times \cancel{T_b}}{4\pi f_c} - 0\right]$$

$$= \sqrt{E_b}\,\frac{2}{\cancel{T_b}} \times \frac{\cancel{T_b}}{2} \Rightarrow \boxed{S_{11} = \sqrt{E_b}}$$

$$\text{111}^{ly} \quad S_{21} = \int_0^{T_b} S_2(t)\,\phi_1(t)\,dt$$

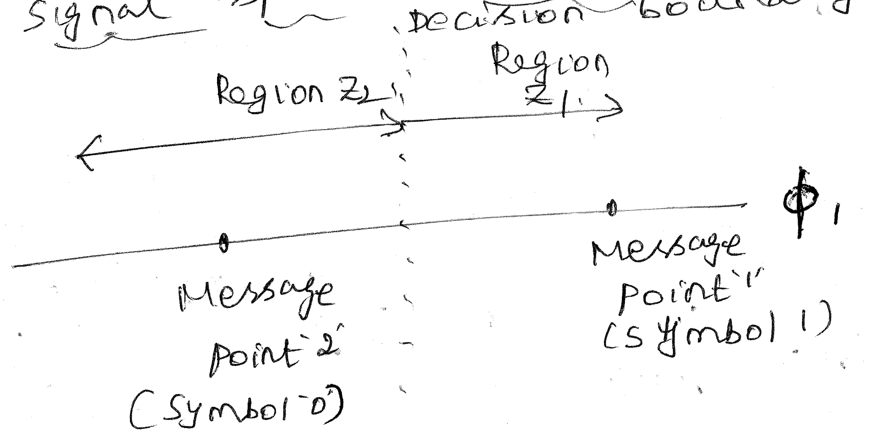
$$\boxed{S_{21} = -\sqrt{E_b}} -$$

∴ Message points are.

$S_1 = [S_{11}] = [\sqrt{E_b}]$

$S_2 = [S_{21}] = [-\sqrt{E_b}]$

Signal space diagram



Decision Rule:

Region Z_1: set of points closest to the message point at $+\sqrt{E_b}$.

Region Z_2: set of points closest to the message point at $-\sqrt{E_b}$.

Decision boundary is constructed in the middle point of the line joining these two message points.

Symbol '1' – if the received signal $x(t)$ falls inside region Z_1

Symbol '0' – if the received signal falls inside region Z_2.

Types of Error: 1. Signal $S_2(t)$ ($S_1(t)$ or) symbol (1/0) was transmitted, due to noise, the received signal point falls in the region (Z_2/Z_1). Hence the receiver decides in favor of symbol (0/1).

The observation scalar x_1 is determined from the received signal $x(t)$ by,

$$\boxed{x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt}.$$

Likelihood function of receiving x_1 when symbol '0' or $s_2(t)$ is transmitted and the receiver decides in favor of symbol '1'.

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right]$$

$$\boxed{f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right]}$$

PROBability of error (Pe) conditional prob. of receiver deciding in favor of '1' given symbol '0'

$$P_e(0) = \int_0^{\infty} f_{x_1}(x_1|0)\,dx_1$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

$dz = \frac{1}{\sqrt{N_0}}\,dx_1$

$dx_1 = \sqrt{N_0}\,dz$

Let $z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$ and changing

if $x_1 = 0 \Rightarrow z = \sqrt{E_b}/\sqrt{N_0}$

the variable of integration from x_1 to z,

$$\therefore P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} \exp(-z^2)\,dz$$

complementary error fn

$\mathrm{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp(-z^2)\,dz$

$\mathrm{erfc}(u) = 1 - \mathrm{erf}(u)$

$$\boxed{P_e(0) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

complementary error function.

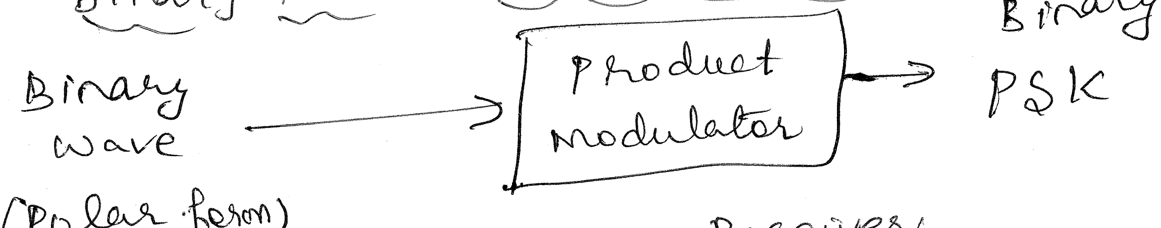
IIIly $P_e(1)$ – conditional prob. of receiver deciding in favor of symbol '0' given '1'

$$\boxed{P_e(1) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

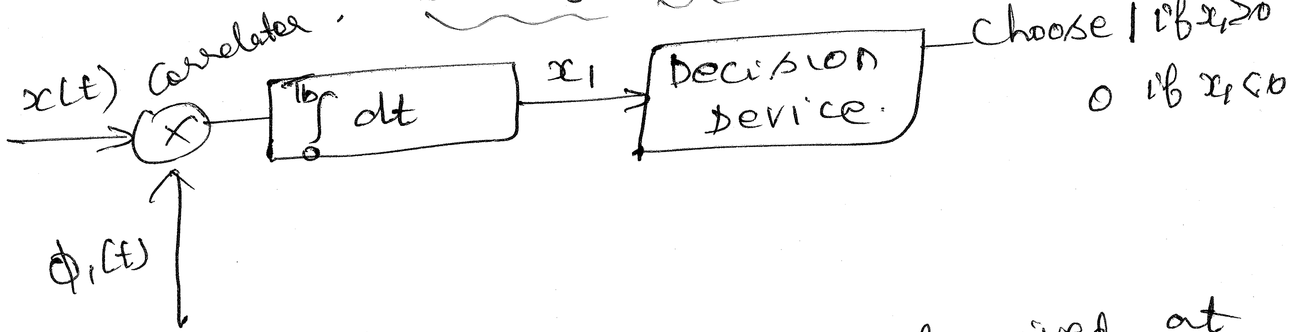
∴ Average error probability of coherent PSK $\boxed{(P_e) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$

Note:- Since the observation space is partitioned in a symmeteric manner, the average prob. of symbol error and symbol error probabilities have the same value.

Binary PSK Transmitter:



Binary PSK Receiver:-



The noisy PSK signal received at the channel output is given to a correlator. Locally generated coherent reference signal $\phi_1(t)$ is also applied to the correlator. The correlator produce a observation scalar x_1 and is compared with a threshold of '0' volts.

If $x_1 > 0$, the receiver decides in favor of symbol '1'

If $x_1 < 0$, the receiver decides in favor of symbol '0'.

Coherent Binary FSK :-

In a binary FSK system, symbol '1' and '0' are represented by one of two sinusoidal waves that differ in freq. by a fixed amount and are defined by.

$$\boxed{S_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t)} \quad 0 \le t \le T_b$$

$$= 0 \quad \text{elsewhere.}$$

where, $i = 1, 2$

$$\boxed{f_i = \frac{n_c + i}{T_b}}$$

Symbol '1' $\quad S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t)$

Symbol '0' $\quad S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_2 t)$

$S_1(t)$ and $S_2(t)$ are orthogonal to each other. but not normalized to have a unit energy.

∴ Orthonormal basis fn is

$$\boxed{\phi_j(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t) \quad 0 \le t \le T_b \qquad = 0. \quad \text{elsewhere}}$$

where $i = 1, 2.$

coefficient $\quad S_{ij} = \int_0^{T_b} S_i(t)\, \phi_j(t)\, dt.$

$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t)\, dt$$

Case 1 :- $i = j$

$$= \frac{2\sqrt{E_b}}{T_b} \int_0^{T_b} \cos^2(2\pi f_i t)\, dt$$

$$\boxed{S_{ij} = \sqrt{E_b}} \quad \text{if } i = j$$

Case 2 $i \ne j$

$$S_{ij} = \frac{2}{T_b}\sqrt{E_b}\int_0^{T_b} \cos(2\pi f_i t)\cos(2\pi f_j t)$$

$$\because \cos A \cos B = \cos(A-B) + \cos(A+B)$$

$$= \frac{2}{T_b}\sqrt{E_b}\left[\int_0^{T_b} \cos 2\pi(f_i - f_j)t\,dt + \int_0^{T_b} \cos 2\pi(f_i+f_j)t\,dt\right]$$

$$= \frac{2}{T_b}\sqrt{E_b}\left[\frac{\sin 2\pi(f_i-f_j)t}{2\pi(f_i-f_j)}\Bigg|_0^{T_b} + \frac{\sin 2\pi(f_i+f_j)t}{2\pi(f_i+f_j)}\Bigg|_0^{T_b}\right]$$

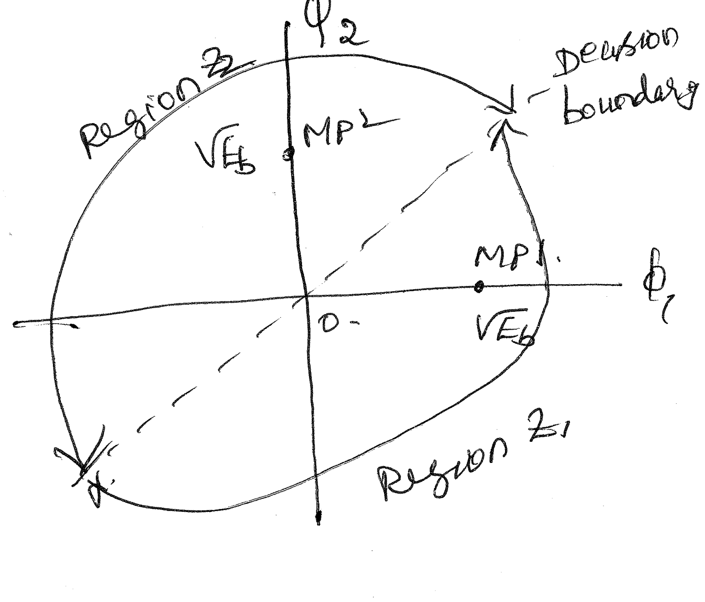
$$\boxed{S_{ij} = 0.}$$

$$\boxed{\begin{aligned}\therefore\ S_{ij} &= \sqrt{E_b} && \text{if } i = j\\ &= 0 && \text{if } i \neq j\end{aligned}}$$

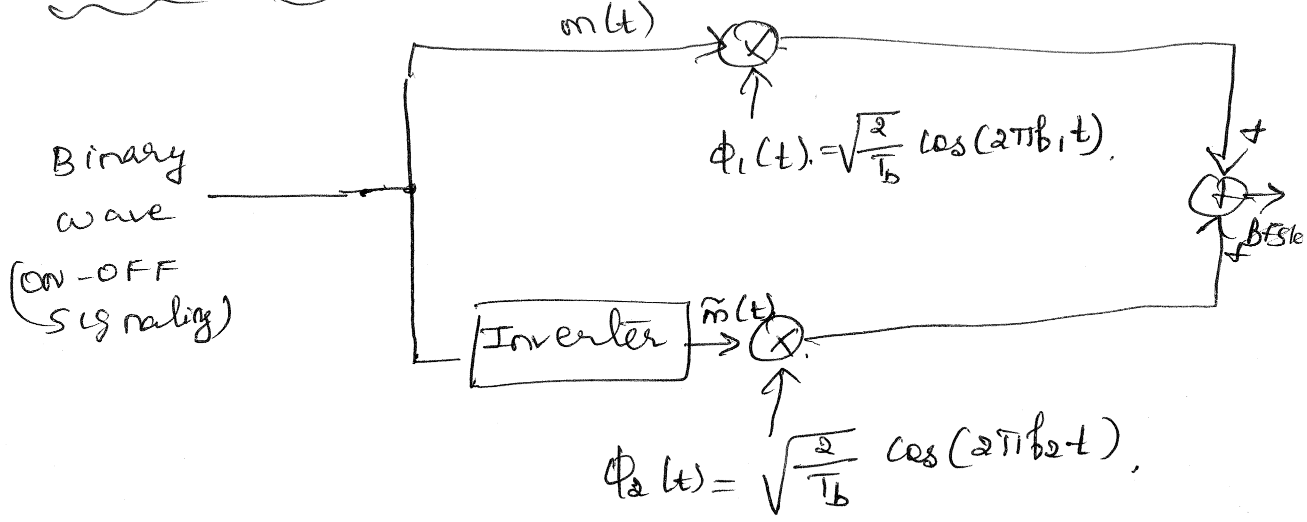
Thus a coherent BFSK system is characterized by having a signal space that is two dimensional ($N=2$) with two message points ($M=2$). The two message points are defined by the two signal vectors.

$$S_{11} = \begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} = \begin{bmatrix}\sqrt{E_b}\\ 0\end{bmatrix}$$

$$S_{22} = \begin{bmatrix}0\\ \sqrt{E_b}\end{bmatrix}$$



BFSK Transmitter:-



The input binary sequence is represented in its ON-OFF form.

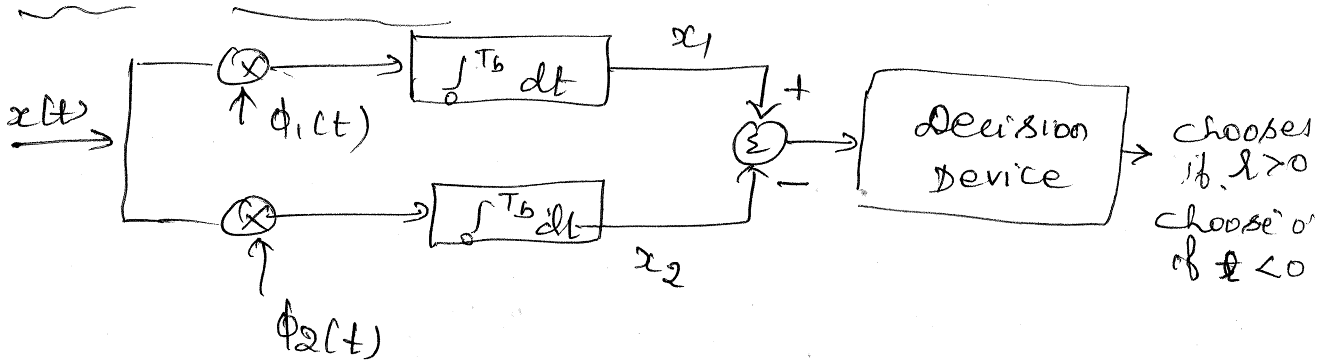
Symbol '1' $\rightarrow$ $+\sqrt{E_b}$ volts

Symbol '0' $\rightarrow$ 0 volts.

Two oscillators are synchronized. For symbol '1' at input, upper channel's oscillator with frequency f_1 is switched ON. while the OSC with frequency f_2 is switched OFF, with the result that freq. f_1 is transmitted

Conversely for symbol '0' upper channel is switched off and lower channel is ON with the result that frequency f_2 is transmitted.

BFSK Receiver:



The noisy received wave $x(t)$ is given as an i/p to the receiver. Receiver consists of two correlators with a common input which are supplied with locally generated coherent reference signals $\phi_1(t)$ and $\phi_2(t)$. The correlator outputs x_1 and x_2 are defined by,

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$x_2 = \int_0^{T_b} x(t)\,\phi_2(t)\,dt$$

Thus the observation vector x has two elements, x_1 and x_2.

The received signal $x(t)$ is

when symbol '1' was transmitted

$$\boxed{x(t) = s_1(t) + w(t)}$$

when symbol '0' was transmitted

$$\boxed{x(t) = s_2(t) + w(t)}$$

where,

$w(t)$ = white Gaussian noise process of

* zero mean.

* power spectral density = $N_0/2$.

Decision Rule :-

* The receiver decides in favor of symbol '1' if $\underbrace{x_1 > x_2}$ (ie) received signal point falls inside the region Z_1

* The receiver decides in favor of symbol '0' if $\underbrace{x_1 < x_2}$; (ie) the received signal point falls inside the region Z_2.

Decision Boundary :- is defined by $\underline{x_1 = x_2}$

Probability of Error :-

Let 'L' is a new gaussion random variable whose sample value 'l' is defined by,

$$\boxed{l = x_1 - x_2}$$

when Symbol '1' was transmitted, the conditional mean of the random variable 'L' is given by,

$$E(L|1) = E[x_1|1] - E[x_2|1]$$
$$= \sqrt{E_b}.$$

when symbol '0' was transmitted, the conditional mean of the RV 'L' is given by,

$$E(L|0) = E[x_1|0] - E[x_2|0]$$
$$= 0 - \sqrt{E_b}.$$

$$\boxed{E[L/0] = -\sqrt{E_b}}$$

variance of RV 'L' is independent of which binary symbol

$$\text{var}[L] = \text{var}[x_1] + \text{var}[x_2]$$

$$\boxed{\text{var}[L] = \frac{N_0}{2} + \frac{N_0}{2} = N_0}$$

conditional prob. density function of RV 'L' equals,

$$f_L(l/0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(l - E(L/0))^2}{2N_0}\right]$$

$$\boxed{f_L(l|0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right]}$$

Conditional probability of error when symbol 0' was transmitted is given by,

$$P_e(0) = \int_0^\infty f_L(l|0)\, dl.$$

$$= \int_0^\infty \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right] dl$$

Let $$z = \frac{l + \sqrt{E_b}}{\sqrt{2N_0}}.$$

$$\therefore\ dz = \frac{dl}{\sqrt{2N_0}} \Rightarrow dl = \sqrt{2N_0}\, dz$$

Lower limit if $l = 0 \Rightarrow z = \sqrt{\frac{E_b}{2N_0}}$

changing the variable of integration (

~~gives~~ from l to z gives,

$$P_e(0) = \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \frac{1}{\sqrt{2\pi N_0}} \exp[-z^2] \sqrt{2N_0}\, dz$$

$$= \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-z^2)\, dz$$

$$= \frac{1}{\sqrt{\pi}} \left(\frac{\sqrt{\pi}}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}} \right) \right).$$

$$\boxed{\because \operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_{u}^{\infty} \exp(-z^2)\, dz}$$

$$\boxed{\therefore P_e(0) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}} \right)}$$

Conditional prob. of error, given symbol 1 was transmitted,

$$\boxed{P_e(1) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}} \right)}$$

$\therefore$ Average prob. of symbol error is given by

$$P_e = (P_e(0) + P_e(1))/2$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}} \right)}$$

Measure of Noise Performance of Digital Modula[tion]

Schemes:-

1) Average Probability of Symbol error (P_e
2) Bit Error Rate (BER).

Relationships between P_e & BER:-

Case 1:- M-ary Modulation Scheme:

$\log_2 M$ bits / symbol.

$$\boxed{BER = \frac{P_e}{\log_2 M}} \qquad M \geq 2.$$

QPSK:- $M = 4$.

$$P_e = \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \Rightarrow BER = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right).$$

M-ary QAM: $M = 16$.

Case 2: If all symbol errors are equally like[ly]

then

$$\boxed{BER = \left[\frac{(M/2)}{(M-1)}\right] P_e}.$$

If M is very large, then BER is limited to

$$\boxed{BER = \frac{P_e}{2}}$$

Ex M-ary FSK.

Bit Error Rate:- is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or bit synchronization perrun...

Measure of Noise Performance of Digital Modulation Schemes:-

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Ex M-ary FSK.

Bit Error Rate:- is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or bit synchronization per unit time (ie) $\boxed{\text{bit errors / unit time}}$

14. (a) QPSK:

The Phase of the Carrier takes on one of four Equally spaced values such as $\pi/4$, $3\pi/4$, $5\pi/4$ & $7\pi/4$ as shown by

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] & 0 \le t \le T \\ 0 & \text{otherwise.} \end{cases}$$

where $i = 1, 2, 3, 4$.

E - transmitted signal Energy per Symbol

T - Symbol duration.

f_c - Carrier frequency.

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

$i = 1, 2, 3, 4$.

The two orthonormal functions are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \quad 0 \le t \le T$$

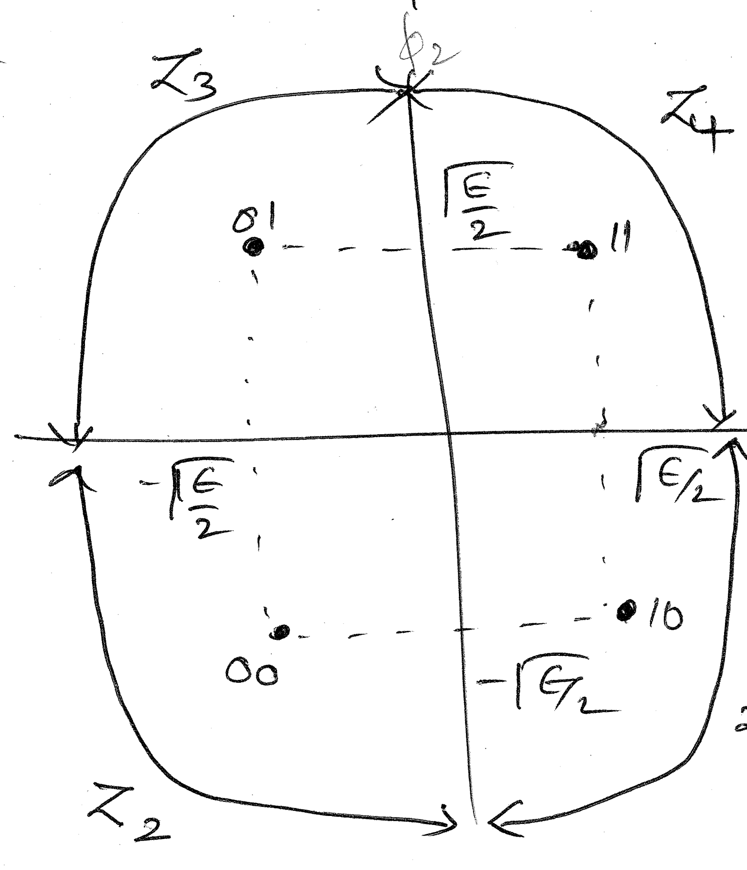
$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \quad 0 \le t \le T$$

Message coordinates are

$$S_i = \begin{bmatrix} S_{i1} \\ S_{i2} \end{bmatrix} = \begin{bmatrix} \sqrt{E} \cos(2i-1)\pi/4 \\ -\sqrt{E} \sin(2i-1)\pi/4 \end{bmatrix} \begin{matrix} \phi_1 \\ \phi_2 \end{matrix} \quad i = 1, 2, 3, 4.$$

QPSK is characterised by a 2D signal space with four message points.

	I/p dibit	Phase of QPSK
S_1	10	$\pi/4$
S_2	00	$3\pi/4$
S_3	01	$5\pi/4$
S_4	11	$7\pi/4$.



coordinates of MP

S_{i1}	S_{i2}
$+\sqrt{E/2}$	$-\sqrt{E/2}$.
$-\sqrt{E/2}$.	$-\sqrt{E/2}$
$-\sqrt{E/2}$	$+\sqrt{E/2}$
$+\sqrt{E/2}$	$+\sqrt{E/2}$.

$\cos(\pi/4) = \sin(\pi/4) = \frac{1}{\sqrt{2}}$

$\cos(3\pi/4) = -1/\sqrt{2} = \sin(3\pi/4)$

$\cos(5\pi/4) = -\frac{1}{\sqrt{2}} = \cos(5\pi/4)$

$\cos(7\pi/4) = \frac{1}{\sqrt{2}}$

$\sin(7\pi/4) = -1/\sqrt{2}$.

The received signal is given by $x(t) = S_i(t) + w(t)$.

$0 \leq t \leq T$.

$i = 1, 2, 3, 4.$

$w(t)$ is the sample function of a Gaussian noise process of zero mean & PSD $N_0/2$

$$x_1 = \int_0^T x(t)\, \phi_1(t)\, dt$$

$$= \sqrt{E} \cos\left[(2i-1)\pi/4\right] + w_1$$

$$x_2 = \int_0^T x(t)\, \phi_2(t)\, dt$$

$$= -\sqrt{E} \sin\left((2i-1)\pi/4\right) + w_2$$

$i = 1, 2, 3, 4.$

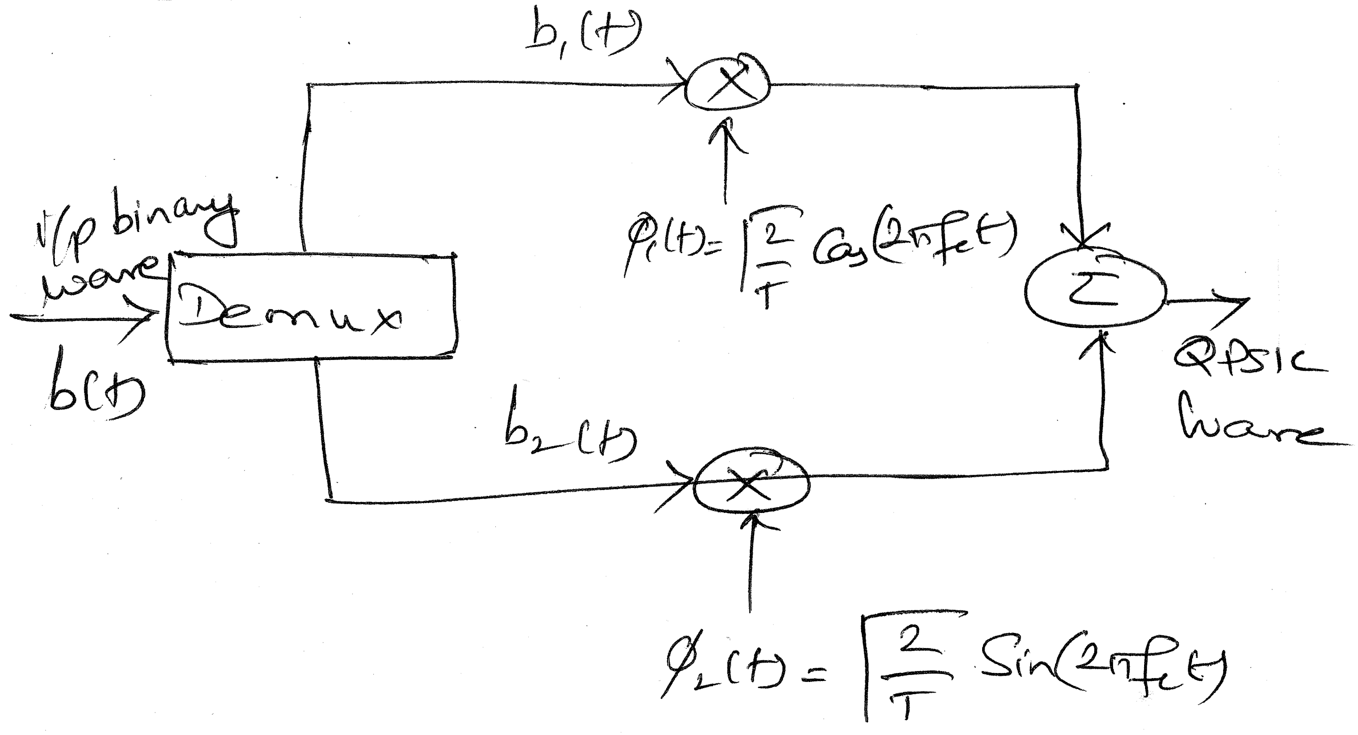
The Prob. of Error is given by

$$P_e = \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

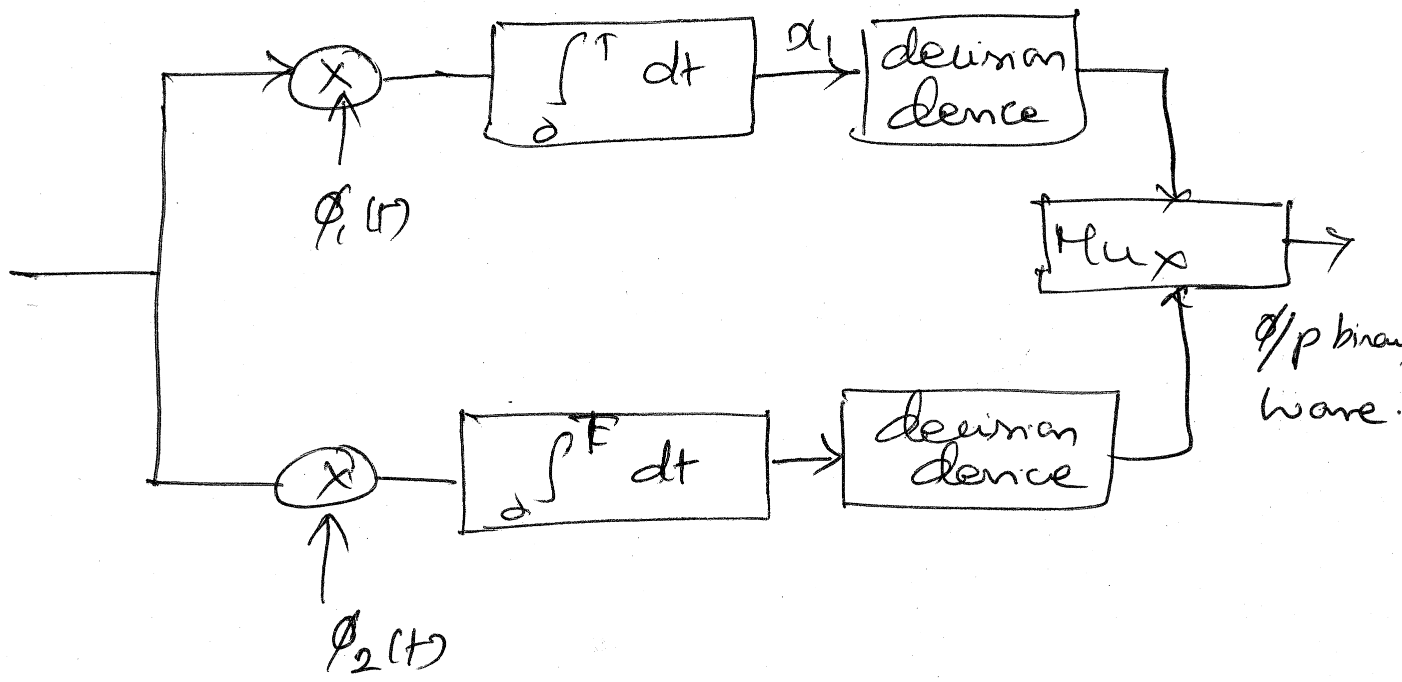
Taken, $E = 2E_b$ we get

$$P_e = \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

QPSK Transmitter:



QPSK Receiver:



x_1 and x_2 are sample values of independent gaussian random variables with mean values

$$\boxed{\begin{aligned} E(x_1) &= \sqrt{E}\cos(2i-1)\pi/4. \\ E(x_2) &= -\sqrt{E}\cos(2i-1)\pi/4. \end{aligned}}$$

the conditional mean is $\sqrt{E/2}$

$var[x] = var[x_1] + var[x_2] = N_0/2 + N_0/2.$

$$\boxed{var[x] = N_0}.$$

Decision rule:-

Case 1:- if observation vector x falls inside the region Z_1 ie ($x_1 > 0$ & $x_2 < 0$) then receiver decides in favor of $s_1(t)$ as 10. probability of correct decision P_c equals the conditional prob. of joint event

Case 2: if x falls inside the region Z_2 (ie) $x_1 < 0$ & $x_2 < 0$ then the receiver decides in favor of $s_2(t)$ as 00.

Case 3: if x falls inside the region Z_3. (ie) $x_1 < 0$ & $x_2 > 0$, then the receiver decides in favor of $s_3(t)$ as 01.

P_c equals joint prob. event

Case 4: if x falls inside Z_4 (ie) $x_1 > 0$ & $x_2 > 0$, then the receiver decides in favor of $s_4(t)$ as 11.

Prob. of correct decision (P_c) equals the product of conditional probabilities of the events $x_1 > 0$ & $x_2 > 0$. The conditional mean of gaussian RV $= \sqrt{E/2}$ & variance $= N_0/2$.

$$\therefore P_c = \left(\int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(x_1 - \sqrt{E/2})^2}{N_0} \right] dx_1 \right) \times$$

$$\left(\int_0^\infty \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(x_2 - \sqrt{E/2})^2}{N_0} dx_2 \right] \right).$$

where,

I integral – conditional prob. of event $x_1 > 0$

II integral – " " " $x_2 > 0$.

Let $\frac{x_1 - \sqrt{E/2}}{\sqrt{N_0}} = \frac{x_2 - \sqrt{E/2}}{\sqrt{N_0}} = z$

~~thus~~ $\frac{dx_1}{\sqrt{N_0}} = \frac{dx_2}{\sqrt{N_0}} = dz$

$$\boxed{\sqrt{N_0}\, dz = dx_1 = dx_2}$$

Lower limit of I & II integral becomes $-\sqrt{E/2N_0}$

Thus, changing the variables of integration from x_1 and x_2 to z, eqn of P_c can be rewritten as,

$$P_c = \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}} \exp\left[-z^2 \right] \sqrt{N_0}\, dz$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\sqrt{\frac{E}{2N_0}}}^{\infty} \exp(-z^2)\, dz = \frac{1}{\sqrt{\pi}} \left[\int_{\sqrt{\frac{E}{2N_0}}}^{0} \exp(-z^2) dz + \int_0^\infty \exp(-z^2) dz \right.$$

$$= \frac{1}{\sqrt{\pi}} \left[-\int_0^{\sqrt{\frac{E}{2N_0}}} \exp(-z^2) dz \quad \sqrt{\pi}/2 \times 1 \right]$$

$$\boxed{P_c = 1 - \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}} \right)}$$

(11)

Error function erf(u) is denoted by,

$$\boxed{erf(u) = \frac{2}{\sqrt{\pi}}\int_0^u \exp(-z^2)\,dz}$$

<u>Properties</u>: $erf(-u) = -erf(u)$

$$\frac{2}{\sqrt{\pi}}\int_0^\infty \exp(-z^2)\,dz = 1$$

Complementary error function.

$$\boxed{erfc(u) = \frac{2}{\sqrt{\pi}}\int_u^\infty \exp(-z^2)\,dz}$$

$$erfc(u) = 1 - erf(u). \Rightarrow erf(u) = 1 - erfc(u)$$

$$\therefore P_c = \frac{1}{\sqrt{\pi}}\left[\int_{-\sqrt{\frac{E}{2N_0}}}^{0} \exp(-z^2)\,dz + \int_0^{\sqrt{\frac{E}{2N_0}}} \exp(-z^2)\,dz + \int_{\sqrt{E/2N_0}}^{\infty} \exp(-z^2)\,dz\right]$$

$$= \frac{1}{\sqrt{\pi}}\left[-\int_0^{-\sqrt{E/2N_0}} \exp(-z^2)\,dz + \frac{\sqrt{\pi}}{2}\,erf\left(\sqrt{\frac{E}{2N_0}}\right) + \sqrt{\frac{\pi}{2}}\,erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right]$$

$$= \frac{1}{\sqrt{\pi}}\left[-\frac{\sqrt{\pi}}{2}\,erf\left(-\sqrt{\frac{E}{2N_0}}\right) + \sqrt{\frac{\pi}{2}}\,erf\left(\sqrt{E/2N_0}\right) + \sqrt{\frac{\pi}{2}}\,erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right]$$

$$= \frac{1}{2}\left[-\left(-erf\left(\sqrt{\frac{E}{2N_0}}\right)\right) + erf\left(\sqrt{\frac{E}{2N_0}}\right) + erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right]$$

$$= \frac{1}{2}\left(2\,erf\left(\sqrt{\frac{E}{2N_0}}\right) + erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right)$$

$$= \frac{1}{2}\left(2\left(1 - erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right) + erfc\left(\sqrt{\frac{E}{2N_0}}\right)\right)$$

$$= \frac{1}{2}\left(2 - 2\,erfc\left(\sqrt{\frac{E}{2N_0}}\right) + erfc\left(\sqrt{E/2N_0}\right)\right)$$

$$\boxed{P_c = 1 - \frac{1}{2}\,erfc\left(\sqrt{\frac{E}{2N_0}}\right)} \Rightarrow \boxed{P_e = erfc\sqrt{\frac{E}{N_0}}}$$

Quadrature Amplitude Modulation: (QAM)

QAM is a special form of hybrid modulation. In QAM, the carrier signal is modulated using both amplitude modulation and phase modulation at the same time.

It is one of M-ary signaling schemes in which 'M' possible signals $s_1(t), s_2(t) \ldots s_M(t)$ can be sent during each signaling interval of duration 'T'.

where $\boxed{M = 2^n \qquad T = nT_b}$ n is an integer, T_b is bit duration.

Adv:- of M-ary over binary signaling scheme

Conserves BW
Efficient channel utilization.

Dis adv:-

Requirement of more power.

The general equation of QAM signal is given as,

$$\boxed{s_i(t) = \sqrt{\frac{2E_b}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_b}{T}}\, b_i \sin(2\pi f_c t)} \quad (1)$$

where,

E_b — Energy of the signal.
a_i & b_i — Independent integers,

Eqn (1) shows that signal $s_i(t)$ consist of two-phase quadrature carriers each of which is modulated by a set of discrete amplitudes, hence named as QAM.

The Signal $s_i(t)$ can be expanded interms of a pair of basis functions:

$$\boxed{\begin{aligned} \phi_1(t) &= \sqrt{\frac{2}{T}} \cos(2\pi f_c t), && 0 \le t \le T. \\ \phi_2(t) &= \sqrt{\frac{2}{T}} \sin(2\pi f_c t) && 0 \le t \le T \end{aligned}}$$

Signal constellation for M-ary QAM consists of a square lattice of message points. The coordinates 'M' Message points are determined using an element $[a_i, b_i]$ of L-by-L Matrix.

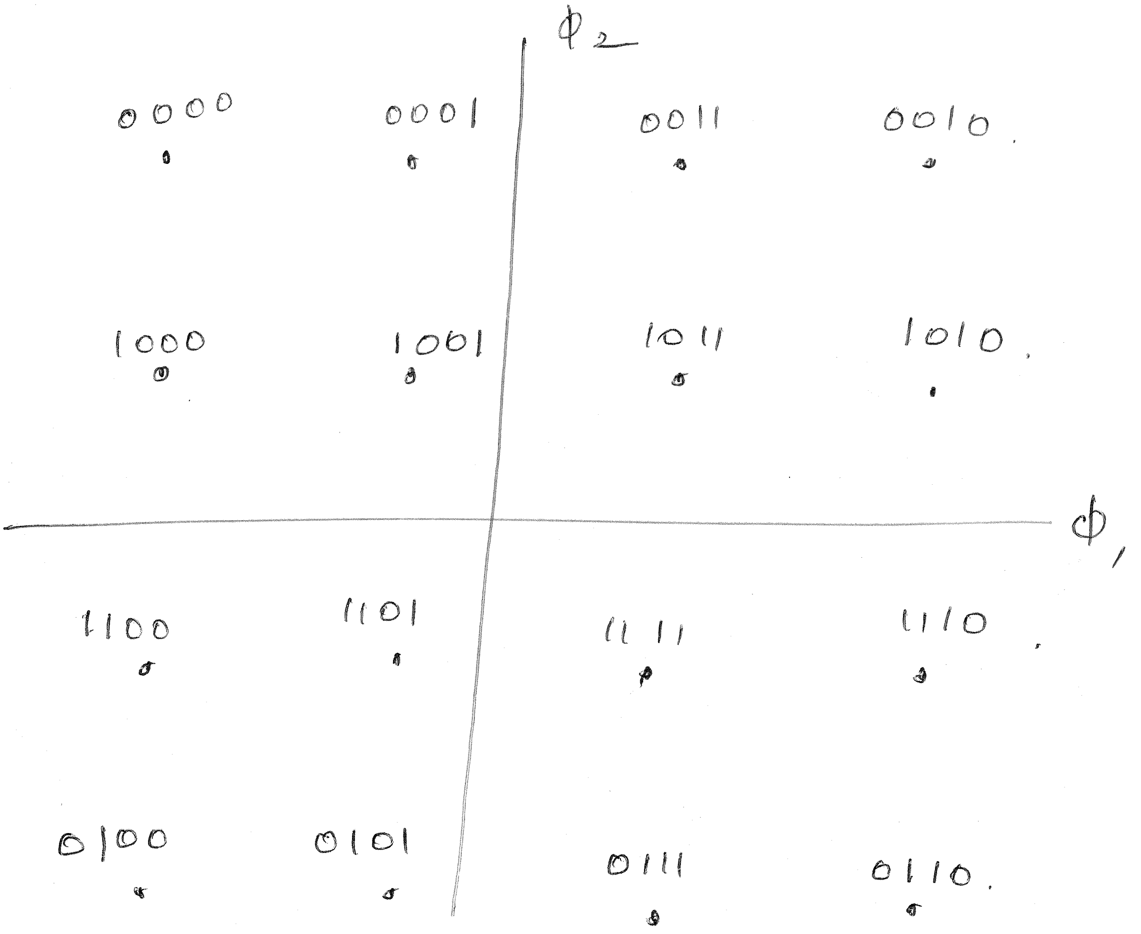
where $\boxed{L = \sqrt{M}.}$

The coordinates of the ith message points are $a_i\sqrt{E}$ and $b_i\sqrt{E_0}$, where (a_i, b_i) is an element of $L \times L$ matrix.

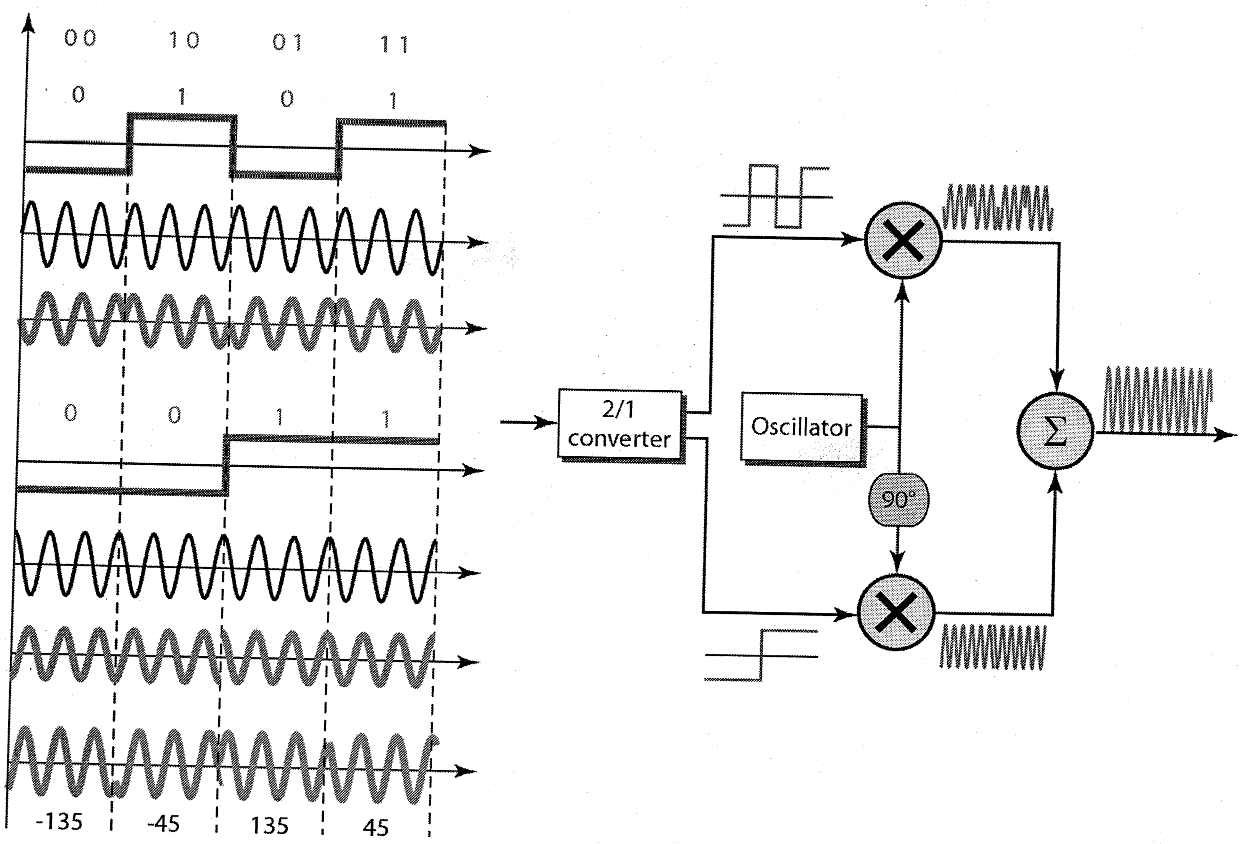
$$\{a_i, b_i\} = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\ \vdots & \vdots & & \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1) \end{bmatrix}$$

$\therefore$ for QAM: $M = 16$; $L = 4$.

$$\{a_i, b_i\} = \begin{bmatrix} (-3, 3) & (-1, 3) & (1, 3) & (3, 3) \\ (-3, 1) & (-1, 1) & (1, 1) & (3, 1) \\ (-3, -1) & (-1, -1) & (1, -1) & (3, -1) \\ (-3, -3) & (-1, -3) & (1, -3) & (3, -3) \end{bmatrix}$$

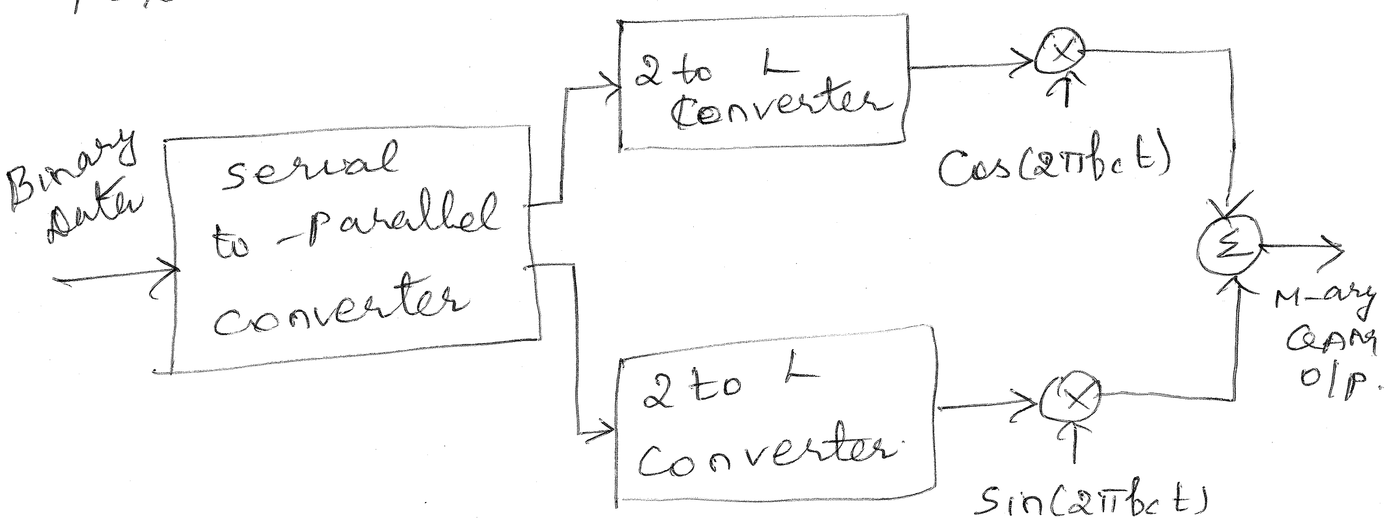


QUADRATURE AMPLITUDE MODULATION

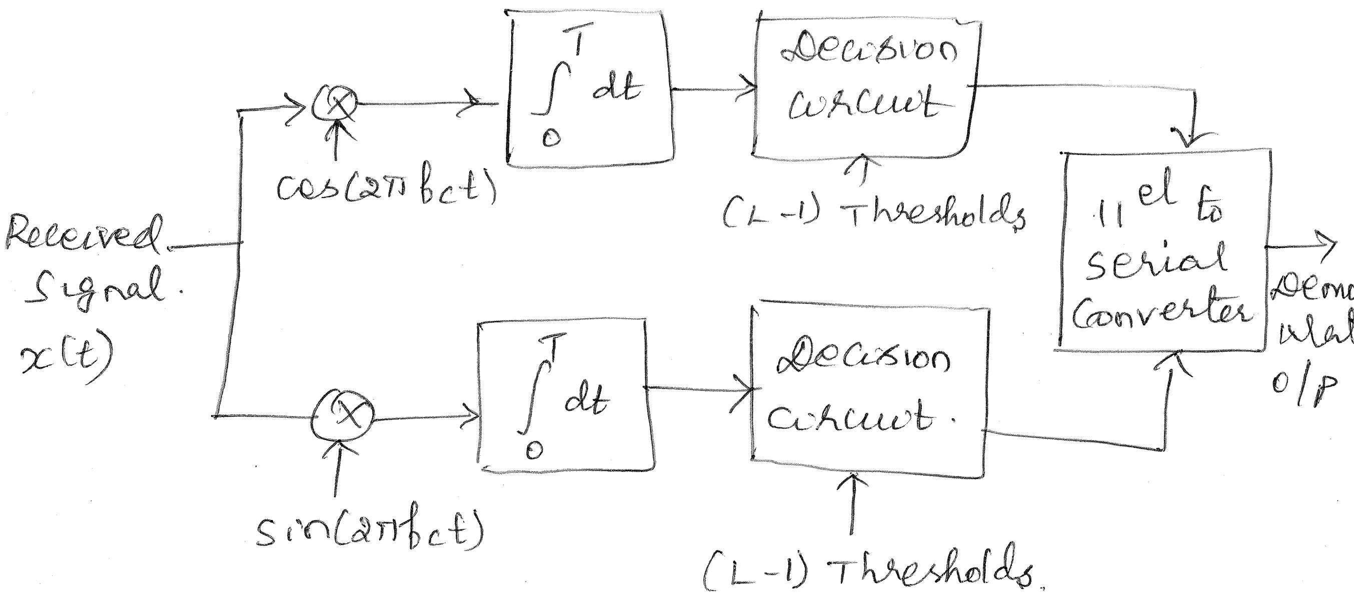


QAM transmitter:

serial-to-parallel converter accepts a binary sequence at a bit rate $\boxed{R_b = 1/T_b}$ and produces two parallel binary sequences whose bit rates are $R_b/2$ each. 2-to-L level converters receives these sequences and then generate polar L-level signals based on the respective in-phase and quadrature channel inputs. The desired QAM signal is then ~~generated~~ produced by multiplexing the generated two polar L-level signals.



QAM Receiver:-



The noisy received signal is given as an i/p to the receiver. Received signals are passed thro' the multiplier and integrator to produce 'L' level signals. The decision circuit compares L-level signals against (L-1) decision thresholds. The two binary sequences so detected are then combined using parallel to serial converter to reproduce the original binary sequence.

Probability of Symbol Error: $\boxed{P_e = (1 - P_c)}$ ——— (1)

where P_c - the probability of correct detection P_c.

$$\boxed{P_c = (1 - P_e')^2} \quad \text{——— (2)}$$

where P_e' is the prob. of symbol error for in-phase and quadrature components of M-ary QAM and are independent components

$$\boxed{P_e' = \left(1 - \frac{1}{L}\right) \operatorname{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)} \quad \text{--- (3)}$$

where $L = \sqrt{M}$

Substituting (2) in (1)

$$P_e = 1 - (1 - P_e')^2$$

$$\boxed{P_e = 2P_e'} \quad \text{--- (4)}$$

Substituting (3) in (4)

$$\boxed{P_e = 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{E_0}{N_0}}\right)} \quad \text{--- (5)}$$

The average value of the transmitted energy can be computed by assuming that the 'L' amplitude levels of the in-phase or quadrature component are equally likely.

$$E_{av} = 2\left[\frac{2E_0}{L} \sum_{i=1}^{L/2} (2i-1)^2\right]$$

$$= \frac{2(L^2-1)E_0}{3}$$

$$\boxed{E_{av} = \frac{2(M-1)E_0}{3} \Rightarrow E_0 = \frac{3E_{av}}{2(M-1)}} \quad \text{--- (6)}$$

Substituting (6) in (5) gives

$$\boxed{P_e = 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left(\sqrt{\frac{3E_{av}}{2(M-1)N_0}}\right)}$$

Synchronization:- is the process of making and maintaining a ~~sync~~ receiver to be synchronous to the transmitter. It is one of the main requirements of coherent detection of digital signals.

Modes of Synchronization:-

1) carrier Synchronization:- is the process of estimating carrier phase and frequency at the coherent detector.

2) Symbol synchronization (or) clock recovery:- It is process of estimating the starting and finishing times of the individual symbols. It helps in determining the sampling time and quenching time of the product integrators.

Carrier Synchronization:- Methods:

1) M-ary loop
2) Squaring loop (M = 2)
3) costas loop

M-ary loop:- It is a straightforward method of carrier synchronization. It modulates a carrier in the data-bearing signal onto the power spectrum of the modulated signal such a way that contains a discrete component at the carrier frequency. Then a narrowband phase locked LOOP (PLL) is used to track this component, thereby providing the desired reference at the receiver. PLL consists of a loop filter, VCO and a multiplier.